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Long-Term Changes in the Rotation of the Earth: 700 B. C. to A. D. 1980 [and Discussion]

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Long-term changes in the rotation of the Earth: 700 B.C. to A.D. 1980

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Occultations of stars by the Moon, and solar and lunar eclipses are analysed for variations in the Earth's rotation over the past 2700 years. Although tidal braking provides the dominant, long-term torque, it is found that the rate of rotation does not decrease uniformly as would be expected if tidal friction were the only mechanism affecting the Earth's rotation. There are also non-tidal changes present that vary on timescales ranging from decades to millennia. The magnitudinal and temporal behaviour of these non-tidal variations are evaluated in this paper.

Introduction

'Now it is most remarkable that observations of transits of Mercury agree with those of the Moon, and with those of the first satellite of Jupiter, in indicating that this apparent inequality (in the Moon's motion) was in part at least due to the Earth's rotation.'

Thus surmised Simon Newcomb just over a hundred years ago (Newcomb 1882) when trying to confirm whether or not the apparent variations in the Moon's orbital motion were due to irregularities in universal time (U.T.). The measure of U.T. is directly proportional to the rate of rotation of the Earth, and hence the positions of the Sun, Moon and planets measured with respect to U.T. will depart from their expected positions by amounts proportional to their mean motions.

Newcomb's results were inconclusive, however. It was only after the Earth's rotation had undergone comparatively large fluctuations in the period 1895 to 1910 that Glauert (1915) was able to show fairly conclusively that the rate of rotation was indeed variable. His result was later confirmed by investigations made by Innes (1925), Brown (1926), de Sitter (1927*a*, *b*) and Spencer Jones (1939).

Clemence (1948) demonstrated how the results of Spencer Jones could be used to replace universal time by a theoretically uniform timescale, later called ephemeris time (E.T.), as the argument of time in the ephemerides of the Sun, Moon and planets. In this transformation, the numerical coefficients in the expression for the mean longitude of the Sun in Newcomb's tables (Newcomb 1895) were left unchanged. Newcomb derived these numerical values from observations made on U.T. mainly in the nineteenth century. Hence the unit of measure of time (now defined as the ephemeris second) in his value for the mean motion of the Sun is approximately equal to the average length of the mean solar second during the nineteenth century. Correspondingly, the ephemeris day is approximately equal to the average length of the mean solar day in the nineteenth century.

In the transformation from U.T. to E.T. in the lunar ephemeris, it was necessary to introduce an empirical value of -22.44 arcsec/cy^2† for the effect of the tidal acceleration in

† In this paper the symbol cy is used to indicate 100 years.

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the mean longitude of the Moon. This was derived by Clemence (1948) from the results of Spencer Jones (1939) and was incorporated in the *Improved lunar ephemeris* (Eckert *et al.* 1954). It is important that an accurate value for the tidal acceleration should be adopted when using observations of the Moon's position to measure secular changes in U.T.; otherwise, the departure of the Moon from its tabular position will be wrongly interpreted as a secular change in U.T. and hence as a secular change in the length of the mean solar day. Despite this drawback, the best resolution of irregularities in U.T. before the introduction of the atomic standards of time-keeping in 1955 is obtained from lunar observations. This is a consequence of both the Moon's rapid motion and the continuous series of observations of its position beginning with the advent of the telescope soon after A.D. 1600.

Brouwer (1952) analysed occultation and meridian circle observations of the Moon and determined ΔT, the difference between E.T. and U.T., at yearly intervals in the period 1820 to 1950. From the first derivative of these results he deduced the variation in the length of the mean solar day relative to the ephemeris day. Fluctuations of several milliseconds on a timescale of decades were noted. These contrasted with the secular (mainly tidal) change of about 2 ms per century which was apparent from ancient observations. The understanding of the physical mechanism which accounts for these decade fluctuations is a topical problem in geophysics (see Lambeck 1980).

Since Brouwer did his investigation, charts giving the topography of the lunar profile have been published by Watts (1963). Timings of occultations of stars at the lunar limb can now be corrected for the uneveness of the limb, which had previously introduced accidental errors as large as ± 6 s and systematic errors of about ± 1 s in deriving ΔT. Further, more observations have been collected from a systematic search of astronomical publications. For these two reasons, it is considered important to re-analyse the occultation data after A.D. 1600 with the object of improving the determination of ΔT. This enables better definition of the magnitude and temporal behaviour of the decade fluctuations in the rotation of the Earth.

As already indicated, ancient observations reveal a systematic increase in the length of the day by some 2 ms/cy which underlies the short-term fluctuations. The relatively large magnitude of the decade fluctuations over the past few centuries makes it impossible to determine any long-term trends in the Earth's rotation from telescopic data; a much longer timescale is needed. Ancient observations must be used for this purpose. These are mainly in the form of eclipses of the Sun and Moon.

The existence of a lunar acceleration was first suspected by Halley (1695). However, it was not until more than half a century later that a quantitative result was obtained by Dunthorne (1749). Both investigators analysed ancient observations of eclipses. Despite tremendous interest in the study of ancient eclipse observations in the nineteenth century, it was not until 1905 that a solar acceleration (apparent, with respect to U.T.) was inferred. From his investigation of ancient eclipse observations – many of which we now know to have been of doubtful reliability – Cowell (1905) concluded that the Sun had a significant secular acceleration. However, he attributed it incorrectly to an acceleration of the Earth's *orbital* motion. It was left to Fotheringham in a series of papers (1908–1921) to put the question of the solar acceleration on a secure foundation.

Defining ν and ν' as the lunar and solar accelerations on U.T., Fotheringham's various analyses gave results for ν' around 3 arcsec/cy^2 (he actually determined $\frac{1}{2}\nu'$). The equivalent rate of lengthening of the day over the past two millennia is close to 2 ms/cy. Fotheringham

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used a wide variety of ancient data in his investigation: eclipses of both Sun and Moon, equinoxes and occultations of stars by the Moon. Although he analysed a number of dubious observations of total and near-total solar eclipses, his remaining data were of fair reliability.

After the work of Fotheringham, the direct analysis of ancient observations attracted little attention for several decades. Geophysicists frequently made use of his results, often rather uncritically, undue attention being paid to his discussion of large solar eclipses. Newton (1970) undertook a major re-analysis of the ancient data, which he later extended to cover mediaeval observations (Newton 1972). He solved simultaneously for the accelerations on E.T. of both the Moon's orbital motion and the Earth's rotation, obtaining a result for the latter which is equivalent to a rate of lengthening of the day since ancient times of 2.4 ± 0.3 ms/cy. However, his investitgations were criticized by Muller & Stephenson (1975) and Muller (1976). Newton introduced into his least squares analyses much data of low weight which, nevertheless, materially affected his final solution. In addition, many of the large solar eclipses which he investigated were subject to the effects of population bias. Muller & Stephenson (1975) and Muller (1976) also attempted to solve simultaneously for the accelerations of the Moon's orbital motion and the Earth's rotation on E.T., but their solutions were questionable for the following reasons. First, there are very few *reliable* ancient observations of solar eclipses – their main data source – that can be used for the purpose of deriving both accelerations. Second, the accelerations of the Moon's motion and the Earth's rotation are highly correlated in the equations of condition.

In the past few years, independent determinations of the Moon's tidal acceleration have been made and these are in fairly good agreement. Hence it is now practicable to adopt a fairly secure value for the acceleration of the Moon's motion and to analyse the ancient eclipse data for long-term changes in the Earth's rotation only. Coupled with this type of approach, the discovery of much hitherto unused ancient observational material of unusually high quality – namely Babylonian timings of lunar eclipses – makes it possible to derive long-term changes in the Earth's rotation with greater confidence than previously. We begin with a discussion of the Moon's tidal acceleration, the constancy of which is crucial to our analysis.

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1. The lunar tidal acceleration $\dot{n}$

All of the observations analysed in the present investigation for changes in the Earth's rotation involve the Moon, either in the form of eclipses or occultations. Hence the accuracy of the lunar ephemeris is important, particularly the acceleration in longitude. Since we lack a satisfactory method of determining this acceleration independently from the data analysed here, it is necessary to consider its derivation by other means. The various techniques used to determine the lunar acceleration involve only the use of modern data (or at the very earliest observations made since A.D. 1700). Consequently, in view of the timescale covered by the material analysed here, the constancy of the acceleration over the historical period is an important question.

The lunar acceleration $\dot{n}$ is directly proportional to the lunar tidal torque and hence to the dissipation of the Earth's rotational energy. Only a small proportion, perhaps 5–10%, of the total dissipation of energy by the tides occurs in the solid Earth and it would appear that the remainder is roughly equally divided between the deep oceans and the continental shelves, but the precise ratio is uncertain (Lambeck 1980). On the timescale of approximately two millennia covered by the data analysed here, only dissipation of these shelf areas might be

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expected to be in any way variable. Studies by Fairbridge (1961) and Mörner (1971) indicate that over this period the global sea-level has never been more than 1 or 2 m above or below the present level. As the average depth of water on the continental shelves is about 100 m and the gradient of the sea-floor increases towards the shore-line, such a change in the global sea level would alter the area of the shelves by less than 1 %. Although there is no model available that predicts just how tidal dissipation would respond to changes in both area and water depth on the continental shelves, the very small magnitude of these same changes suggests that the effect on global dissipation will be negligible. The assumption of a constant value for $\dot{n}$ over the historical period thus seems justified.

In recent years a variety of techniques has been used to estimate the value of $\dot{n}$. These include lunar laser ranging, analysis of the orbits of artificial satellites, numerical tidal models and analysis of astronomical observations. Individual observations are summarized in table 1, which also cites appropriate references.

Table 1. Recent determinations of the lunar tidal acceleration, $\dot{n}$

method	reference	$\dot{n}$ / arcsec/cy^2
transits of Mercury	Morrison & Ward (1975)	-26 ± 2
numerical tidal model	Lambeck (1980, p. 335)	-29.6 ± 3.1
artificial satellites	Cazenave (1982)	-26.1 ± 2.9
lunar laser ranging	Dickey & Williams (1982)	-25.1 ± 1.2

The agreement in table 1 between results obtained with a wide variety of methods is very satisfactory. In the present investigation we have adopted the result of Morrison & Ward (1975). Their investigation covered by far the longest timescale (some 250 years) and the value which they derived, i.e. $-26''/\mathrm{cy}^2$, happens to be identical to the mean of the results listed in table 1. However, if a different value for $\dot{n}$ is adopted in the future, our results can be readily adjusted to conform to it.

2. Observations

The data analysed in this paper can be conveniently divided into three distinct chronological groups, the boundaries of which are not entirely arbitrary. The date ranges are as follows: *ca.* 700 B.C. to A.D. 500; A.D. 500 to 1600; A.D. 1600 to 1980. These three groups will be treated in reverse chronological sequence as this is the order which has proved most convenient for analysis. The data since 1861 have already been considered in some detail by Morrison (1979*a*); however, the pre-telescopic observations in particular require a number of comments.

2.1. *The period* A.D. 1600–1980

Most of the observations analysed in this period are of occultations of stars by the Moon. These are published in the catalogues of Morrison *et al.* (1981) for the years 1623–1942 and Morrison (1978) for 1943–1971.

Beginning at 1955 July 1, the values of ΔT are derived from

$$\Delta T = (\mathrm{T.A.I.} - \mathrm{U.T.1}) + 32.184\ \mathrm{s},$$

where T.A.I. represents the international atomic timescale and U.T.1 is a precise measure of U.T. freed from the effects of polar motion. The constant offset of 32.184 s is defined in I.A.U.

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(1977). The values of T.A.I. − U.T.1 for the years 1956–1980 were taken from the Annual Reports of the Bureau International de l'Heure (B.I.H.).

The total number of occultations analysed in the period from 1861 to 1955 was about 50000. The corresponding number for the period between 1623 and 1860 was about 2000. A histogram showing the annual distribution of the observations in this latter period is shown in figure 1.

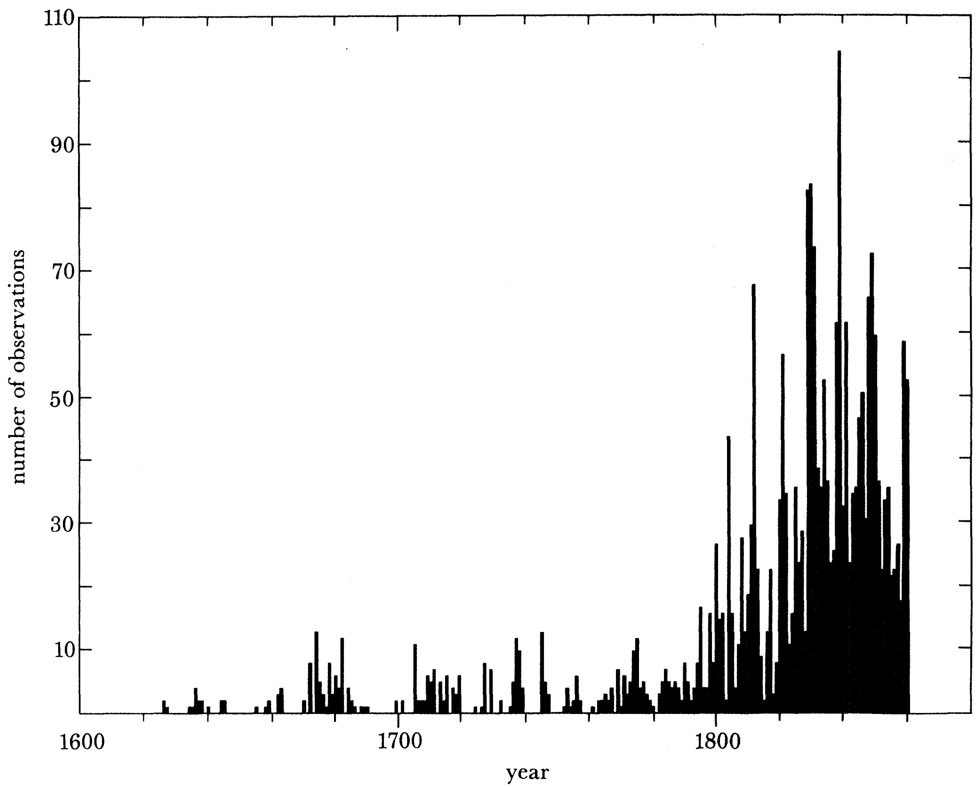

FIGURE 1. Distribution by year of occultation observations in the period A.D. 1620 to 1860.

There were no occultation data of sufficient accuracy for the present purpose between A.D. 1600 and 1622. It was not until the use of the telescope became fairly widespread that the timing of occultations was achieved with any real accuracy: say ± 1 min. Until about 1670, the use of star altitudes as the principal method of timing such observations severely limited their accuracy. Hence in this early telescopic period, timings of the last contacts of solar eclipses were also included even though eclipse contacts are in general not sharp events. The eclipse timings in the period 1621–1806 are published by Morrison *et al.* (1981). About 140 of these timings made in the period 1621–1670 are used in this analysis. As with occultations, there are no usable eclipse observations in the first two decades of the seventeenth century.

All the times of the observations were reduced where necessary to U.T. from the original time system or altitude measurement.

2.2. *Mediaeval observations*: A.D. 500–1600

We have chosen the epoch A.D. 500 as a convenient dividing point between what might be termed the 'ancient' and 'mediaeval' periods. As there are very few useful observations between about A.D. 100 and 800, precise demarcation is unimportant. In both the present section and

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the following (2.3), we shall concentrate on eclipses, since there are no other observations of comparable frequency and precision. A full list of all the pre-telescopic (both ancient and mediaeval) observations discussed here will be published elsewhere.

Useful mediaeval observations fall into two distinct categories. These are: untimed sightings of total and near-total solar eclipses recorded in a variety of sources; and a series of timed solar and lunar eclipse contacts recorded by Arab astronomers between about A.D. 800 and 1000. We shall consider each of these data sets in turn.

2.2.1. *Untimed observations of total and near-total solar eclipses*

Untimed observations of solar eclipses in this period are largely reported in non-astronomical works, notably mediaeval European monastic chronicles. Modern observations of total eclipses of the Sun indicate that the unaided eye is able to decide very precisely just when an eclipse becomes total. From the time of Halley (1715), there have been several very successful attempts to fix accurately the edges of zones of totality on the Earth's surface by unaided eye observation.

Numerous mediaeval accounts of large solar eclipses are on record (see Newton (1972); Stephenson (1972); Muller (1975) for detailed translations). In these descriptions, the appearance of stars and the allusion to darkness are commonplace. However, such phenomena in themselves are not reliable indications of totality; in particular, stars are occasionally reported during eclipses which were only annular. However, more than 30 mediaeval observations specifically mention the complete disappearance of the Sun and here the record deserves considerable weight.

Several mediaeval eclipse records expressly deny totality and these are just as valuable as total eclipse observations. However, descriptions that relate to annular eclipses of the Sun at this period are usually quite vague. The accurate observation of an annular eclipse with the unaided eye requires a fair degree of skill since part of the Sun remains visible at all times. Accordingly, we have included in the present investigation only two observations of annular eclipses; both careful descriptions by Arab astronomers.

Important questions concern the reliability of both the recorded date and the inferred place of observation for each eclipse. We shall consider these aspects in turn. Fortunately, for all of the observations we have analysed, the date is fairly secure. As a general rule, when a mediaeval work (particularly a chronicle) takes the trouble to provide what amounts to an astronomically usable record of an eclipse, the precise date is normally given. Most European dates are expressed directly in terms of the Julian calendar. In other cases (e.g. Chinese and Arab dates), readily available conversion tables have been employed to reduce the recorded date to the Julian calendar. For almost every selected observation, the historical date agrees *exactly* (to the day) with the computed date of a solar eclipse. In a few instances, only the year is specified, but with such a small uncertainty there is no difficulty in isolating the true date; total or near-total eclipses are extremely rare at any given place.

The precise place of observation for nearly all of the eclipses used here is well established. Most of these events were recorded in monastic or town chronicles which were largely concerned with very local occurrences. The eclipse records in such works are often both vivid and highly original, suggesting that the annalist was writing from personal recollection. In China, the expected place of observation is normally the capital. We have concentrated on those eclipse records contained in the astronomical treatises of the various dynastic histories, and these are, in general, based on the observations of the court astronomers, located at the capital. It was

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the duty of the astronomers to keep a regular watch of the sky for any omens which might occur and when necessary present an official report to the emperor. Only rarely do we find an assertion that an observation was reported from the provinces, for example when cloudy weather prevailed at the capital.

2.2.2. *Timed observations of lunar and solar eclipses*

In 1878 Newcomb brought to the attention of the astronomical world a series of timings of both solar and lunar eclipses made by Arab astronomers between about A.D. 800 and 1000. His source was Caussin (1804). First at Baghdad, and later (after about A.D. 950) at Cairo, astronomers such as Ibn Iunis carefully timed 25 eclipses. Each entry states the precise place of observation (i.e. Baghdad or Cairo). As Newcomb showed, almost all of the recorded dates as converted to the Julian calendar by Caussin in his translation of the original texts agree exactly with the dates of calculated eclipses (e.g. as referenced by Oppolzer 1887). Errors are never more than a single day. Difficulties relate mainly to the rather low precision of measurement. In addition, there are several obvious errors of transcription.

Almost all of the observations were altitude measurements rather than direct timings. For solar eclipses, the altitude of the Sun at the appropriate contact (first or last: there were no total eclipses in the list) was usually measured. In the case of lunar eclipses, it was the practice of the Baghdad astronomers to measure the altitude of a suitably placed bright clock star when the contact occurred. The Cairo observers, however, usually preferred to take altitudes of the Moon itself. Most measurements were expressed to the nearest degree, although higher precision (normally $\frac{1}{2}$ or $\frac{1}{3}$ deg) was occasionally attempted. However, we have felt it unwise to assign different weights to individual observations. The likely error in each altitude determination implies a timing error of some 5 min.

2.3. *Ancient observations*: *ca.* 700 B.C. *to* A.D. 500

Ancient eclipses, which we have found to be of value in the study of the Earth's rotation, fall into three main groups: (1) untimed sightings of total and near-total solar eclipses (mainly from China); (2) a large number (roughly 40) of accurately timed contacts of lunar eclipses recorded by Babylonian astronomers between about 700 B.C. and 50 B.C.; (3) further untimed Babylonian observations of lunar eclipses from this same period for which the Moon was stated to have risen or set while it was partly or wholly obscured. We shall discuss data in each category in turn.

2.3.1. *Untimed sightings of total and near-total solar eclipses*

Untimed sightings of total and near-total solar eclipses have been the focus of many investigations for over 150 years. The best known analysis of such material is probably that of Fotheringham (1920). However, in a recent paper (Stephenson & Morrison 1982) we have shown that such was the unreliability of the historical sources investigated by Fotheringham, that in no case could it be established with any degree of confidence that the date, place *and* magnitude were all known. Fotheringham mainly abstracted from Greek and Roman classical sources, merely because he lacked good material from elsewhere. Following Muller & Stephenson (1975), we have preferred to use Chinese observations (five sightings of total and near-total solar eclipses between 198 B.C. and A.D. 120) together with a single Babylonian report of a total solar eclipse in 136 B.C.

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2.3.2. *Late Babylonian timed observations of lunar eclipses*

The Late Babylonian astronomical texts which were discovered in the ruins of Babylon about a century ago (see Sachs 1974), most of which are now in the British Museum, number some 1500 fragments in all. Careful drawings of most of these have been published by Sachs (1955), but so far no systematic translation has been made. The most fundamental texts are astronomical diaries, which give a month by month account of observations of the more important lunar, solar and planetary phenomena. Individual tablets typically covered six months of systematic observation, but because of extensive damage when they were dug up, most fragments are rather small. Around 300 B.C. it had become the practice for the Babylonian astronomers to abstract material from the diaries for the preparation of lists of eclipses, planetary phenomena, etc. mainly to assist in the prediction of similar events in the future. Nearly all of the extant eclipse observations are recorded on these latter texts and there is a heavy bias towards lunar eclipses in the preserved samples; scarcely any solar eclipse observations are usable.

Most of the eclipse records to be found on the Babylonian texts were translated and edited a few years ago by P. J. Huber of Harvard University in an unpublished memoir. One of us (F.R.S.) on a recent visit to the British Museum checked all important readings (particularly numbers).

All of the surviving texts are damaged and to date less than 10% of the original material is accounted for. From the point of view of dating, the astronomical diaries have suffered most since they usually carried a reference to the year only once (as the first entry). Using planetary data preserved on some of the fragments, Sachs (1955) was able to date a number of diaries for which the recorded year was missing. However, many such fragments are undatable. Fortunately, texts specifically devoted to eclipses present relatively few dating problems. On such texts, every appropriate king's name is specified in its relevant place and additional historical details are often inserted. Hence even extensively damaged tablets can usually be dated without recourse to astronomical calculation.

The most systematic eclipse observations on the Babylonian astronomical texts relate to the times of first contact for lunar eclipses. From sometime before 700 B.C. – precisely when is unknown – it was the practice to time the beginning of an eclipse in relation to either sunrise or sunset, whichever was nearer.

Little is known about the method of timing adopted, but some type of clepsydra (water clock) was probably used. The timing of first contacts of eclipses was evidently facilitated by the use of rough predictions which would forewarn intending observers. The unit of time was the *uš*, the time taken for the celestial sphere to rotate through one degree, i.e. 4 min. Over the seven centuries or so covered by the extant texts, a gradual improvement in the precision of timing can be traced. With few exceptions, the very earliest observations are timed only to the nearest 10 *uš*. Around 610 B.C. the typical accuracy improved to 5 *uš*. Finally, after about 550 B.C. all observations are quoted to the nearest *uš*. The recorded intervals of time vary from as small as 3 *uš* to over 100 *uš*. In particular, for 8 observations the measured interval is an hour (15 *uš*) or less. Even taking into account the effect of clock drift, such data are obviously of great value since at this early period ΔT is of the order of several hours.

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2.3.3. *Babylonian observations in which the Moon was seen to rise or set while eclipsed*

More than 20 Babylonian records state that the Moon rose or set while it was either partly or totally eclipsed. Such a phenomenon is by no means uncommon and it would be fairly easy to observe accurately since the full Moon is readily visible on the horizon when the sky is clear. In principle, the mere fact that at the moment of moonrise or moonset part of the lunar disc was seen to be in shadow enables a rather precise upper or lower bound to be placed on the value of ΔT without the need for an accurate timing. However, in practice, the rather long duration of a typical lunar eclipse (a few hours) restricts the use of such data. Fortunately, in a few cases the text indicates that the Moon happened to rise or set fairly close to one of the umbral contacts.

3. Method of calculation of ΔT and change in l.o.d. (length of day)

The reduction of telescopic observations of occultations of stars by the Moon is discussed in detail by Morrison (1979*b*) and will not be repeated here. For the few seventeenth century timed eclipse contacts, used to supplement the occultation data at this period, the method of analysis follows essentially that outlined in section 9D of the *Explanatory supplement* (1974). In this section we shall concentrate on the reduction of the pre-telescopic material which is in the form of both timed and untimed eclipses.

We suppose that over hundreds of years torques or changes in the moment of inertia of the Earth accelerate the Earth's rotation by an average amount $\dot{\omega}$. This will cause apparent accelerations ν' and ν in the mean motions of the Sun and Moon with respect to U.T.:

$$\nu' = -n'(\dot{\omega}/\omega), \tag{1}$$

$$\nu = \dot{n} - n(\dot{\omega}/\omega), \tag{2}$$

where

n' is the mean motion of Sun $= 0.130 \times 10^9 {}''/\text{cy}$
n is the mean motion of Moon $= 1.733 \times 10^9 {}''/\text{cy}$
ω is the rate of rotation of Earth $= 47.47 \times 10^9 {}''/\text{cy}$
$\dot{n}$ is the tidal acceleration of Moon with respect to a uniform timescale.

Observations of eclipses provide data on the observed separation of the Sun and Moon as seen from a point on the Earth. Let L' and L be the mean longitudes of the Sun and Moon for the time and place of an eclipse calculated from the gravitational theories of their motions but not including the tidal acceleration $\dot{n}$ (Only the date is sufficient for a total solar eclipse; the observation of totality itself obviates the need for an accurate time.) In principle, from an observation of an eclipse – whether of Sun or Moon – we obtain:

$$\nu - \nu' = 2(L - L')/t^2, \tag{3}$$

where t denotes time measured in centuries from some fixed epoch in modern times (see §5.1).

From (1) and (2) we find

$$(\dot{\omega}/\omega) = \{\dot{n} - (\nu - \nu')\}/(n - n'). \tag{4}$$

In practice, a value of $\dot{n}$ $(-26''/\text{cy}^2)$ is assumed in calculating the lunar mean longitude, and rather than deduce the difference in mean longitude $L - L'$, the expected time of the eclipse

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is calculated on the time-argument of the ephemerides, terrestrial dynamical time (TDT). The basic method of calculation is set out in section 9D (timed contacts for solar eclipses) and 9E (timed contacts for lunar eclipses) of the *Explanatory supplement*. TDT is closely related to its precursor, E.T., but the origin and unit of measure on the TDT scale are defined (I.A.U. 1977) in relation to T.A.I. At the epoch 1977 January 1, 0 h T.A.I.

$$\mathrm{TDT} = \mathrm{T.A.I.} + 32.184\ \mathrm{s},$$

and the length of the day on the TDT scale at this epoch is 86400 SI seconds exactly. The difference between TDT and the observed time in U.T. is denoted by ΔT. It is the cumulative error in time of a clock keeping U.T.

The foregoing is mainly directed towards timed eclipse observations. The analysis of a lunar eclipse observation in which the Moon was seen to rise and set while eclipsed is similar to that for a timed lunar eclipse. It is necessary in addition to compute the U.T. of moonrise or moonset to provide a reference point; note that these computed times are only weakly dependent on the assumed value of $\dot{n}$. For untimed observations of total and near-total solar eclipses the track of totality (or annularity) in ephemeris longitude is calculated as set out in section 9C of the *Explanatory supplement*. The difference between the ephemeris longitude and the longitude from which the eclipse was observed is equal to ΔT.

Assuming that there is a constant net acceleration over the whole or part of the period of this analysis, the cumulative discrepancy in time is given by

$$\mathrm{TDT} - \mathrm{U.T.} = \Delta T = ct^2 \tag{5}$$

where c is a constant in units of seconds per century per century and t is reckoned in centuries from the epoch at which the length of the U.T. day (i.e. the mean solar day) is equal to the fixed length (86400 SI seconds) of the reference day of the TDT scale.

The relationships between the time-derivatives of ΔT and the l.o.d. are set out in table 2.

TABLE 2. RELATION BETWEEN DERIVATIVES OF ΔT, L.O.D. AND RATE OF ROTATION

description	quantity	unit
time	t	cy
acceleration parameter	c	s/cy^2
cumulative discrepancy in time	$\Delta T = ct^2$	s
≡ sidereal displacement angle	$\Delta\theta = -ct^2(1.00274 \times 15/206265)$	rad
	$= -7.29 \times 10^{-5}\Delta T$	rad
first derivative of ΔT	$\mathrm{d}(\Delta T)/\mathrm{d}t = 2ct$	s/cy
≡ excess length of day	$\Delta\,\mathrm{l.o.d.} = 2ct(1000/36525) = 0.0548ct$	ms/d†
≡ change in rate of rotation	$\omega - \omega_0 = \Delta\omega = -2ct(1.00274 \times 15/(206265 \times 36525 \times 86400))$	rad/s
	$= -4.62 \times 10^{-14}ct$	rad/s
	$= -0.843 \times 10^{-12}\Delta\,\mathrm{l.o.d.}$	rad/s
second derivative of ΔT	$\mathrm{d}^2(\Delta T)/\mathrm{d}t^2 = 2c$	s/cy^2
≡ rate of change of l.o.d.	$\mathrm{d(l.o.d.)}/\mathrm{d}t = 2c(1000/36525) = 0.0548c$	ms/d/cy‡
≡ fractional rate of change in rate of rotation	$(\dot{\omega}/\omega) = -2c/(86400 \times 36525 \times 100)$	a^{-1}
	$= -0.634 \times 10^{-11}c$	a^{-1}
	$= -11.6 \times 10^{-11}\,\mathrm{d(l.o.d.)}/\mathrm{d}t$	a^{-1}
≡ acceleration	$\dot{\omega} = -2c(1.00274 \times 2\pi/86400)/(86400 \times 36525)^2$	rad/s^2
	$= -1.46 \times 10^{-23}c$	rad/s^2
	$= -2.67 \times 10^{-22}\,\mathrm{d(l.o.d.)}/\mathrm{d}t$	rad/s^2
reference l.o.d.:	$(\mathrm{l.o.d.})_0 = 86400$ SI s exactly	s
corresponding rate of rotation	$\omega_0 = 1.002737811906(2\pi/86400)$	rad/s

† Usually written ms.
‡ Usually written ms/cy.

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5. Results for ΔT

In the computation of lunar positions, the ephemeris used was $j = 2$ (I.A.U. 1968). This ephemeris is based on a value for $\dot{n}$ of $-22.44''/\text{cy}^2$. In order to incorporate fully our choice of $\dot{n}$ (i.e. $-26''/\text{cy}^2$), it was necessary to apply the following additions to the mean lunar longitude of the ephemeris:

$$\Delta L = -1.54'' + 2.33''T - 1.78''T^2 \tag{6}$$

where T is measured in centuries from the epoch 1900. For details see Morrison (1979*b*).

5.1. *The telescopic period*: A.D. 1620–1980

As explained earlier (§2.1), ΔT values for the interval between 1955 and 1980 were obtained from T.A.I.–U.T.1. In the period 1861–1954 there were ample occultations to enable independent annual means for ΔT to be derived. However, before 1861 it was necessary to smooth individual ΔT values by the use of cubic splines with 13 knots. A discussion of the various techniques is given by Morrison (1979*a*) and Morrison & Stephenson (1981). Average values of ΔT at the beginning of each year (every five years between 1630 and 1780) are given in table 3. For the earliest period (up to 1780), results in table 3 are rounded to the nearest second, while between 1780 and 1859 they are expressed to the nearest 0.1 s. Since 1860, the values of ΔT are quoted to the nearest 0.01 s.

The individual results for ΔT derived from timings of occultations and solar eclipses (4th contacts only) over the interval from A.D. 1620 to 1860 are plotted in figure 2*a*. In this diagram, crosses represent occultations and open circles eclipses (up to A.D. 1670 only). Although the scatter in the data during the seventeenth century is high, there is a definite upward trend in the fitted curve; this trend is apparent from both the occultation and eclipse results.

Figure 2*b* shows the same data as figure 2*a*, but on a larger scale. It is evident that after about A.D. 1780 there are sufficient data to reveal some detail of the fluctuations in ΔT.

Figure 3 continues the ΔT plot from 1861 to 1980. Each point represents an annual mean. The values obtained by Brouwer (1952), adjusted to $\dot{n} = -26''/\text{cy}^2$, are joined by dotted lines. Brouwer's analysis appeared to reveal erratic fluctuations of short period in the Earth's rotation. However, our use of a larger number of data and the application of lunar limb profiles in the analysis of each individual timing has resulted in a considerably smoother curve.

Particularly large fluctuations in the Earth's rotation occurred in the period from 1861 to 1900. Figure 4 illustrates the accuracy of the individual annual means for ΔT over this time-interval. As before, the values derived by Brouwer are shown for comparison.

The ΔT curve for the entire telescopic period is shown in figure 5. This reveals the general parabolic shape, in agreement with relation (5) §3. As discussed later (§5.2), Arabian observations around A.D. 950 give the following average result over the past millennium:

$$\Delta T = 25.5t^2 \text{ s.}$$

In figure 5, this parabola (dashed) is superposed on the telescopic results, and it is seen to be a reasonable fit when t is measured from about A.D. 1820 (the precise year is almost independent of the coefficient of t^2 for the parabola). In the absence of fluctuations in ΔT, A.D. 1820 is thus the approximate year at which the length of the U.T. day is equal to the fixed reference day of 86400 SI seconds. It is not possible to fix the true epoch precisely because the choice is

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Table 3. Mean values of ΔT and Δ l.o.d. for the telescopic period

Changes in the length of the day are measured relative to the standard day of 86400 SI seconds. Values tabulated relate to the beginning of each year (or 5 year period up to A.D. 1780).

year	ΔT/s	Δl.o.d./ms	year	ΔT/s	Δl.o.d./ms	year	ΔT/s	Δl.o.d./ms
1630	85	−8	1801	13.4	−0.8	1852	7.3	0.3
1635	72	−6	1802	13.1	−0.7	1853	7.4	0.3
1640	62	−5	1803	12.9	−0.5	1854	7.5	0.2
1645	54	−4	1804	12.7	−0.4	1855	7.6	0.2
1650	48	−3	1805	12.6	−0.3	1856	7.7	0.2
1655	43	−3	1806	12.5	−0.2	1857	7.7	0.2
1660	37	−3	1807	12.5	−0.1	1858	7.8	0.1
1665	32	−3	1808	12.5	0.0	1859	7.8	0.1
1670	26	−3	1809	12.5	0.0	1860	7.88	−0.15
1675	21	−3	1810	12.5	0.0	1861	7.82	−0.57
1680	16	−3	1811	12.5	0.1	1862	7.54	−1.04
1685	12	−2	1812	12.5	0.0	1863	6.97	−1.30
1690	10	−1	1813	12.5	0.0	1864	6.40	−1.43
1695	9	0	1814	12.5	0.0	1865	6.02	−1.84
1700	9	0	1815	12.5	−0.1	1866	5.41	−2.43
1705	9	0	1816	12.5	−0.1	1867	4.10	−2.98
1710	10	0	1817	12.4	−0.2	1868	2.92	−2.71
1715	10	0	1818	12.3	−0.3	1869	1.82	−2.55
1720	11	0	1819	12.2	−0.5	1870	1.61	−2.63
1725	11	0	1820	12.0	−0.6	1871	0.10	−2.42
1730	11	0	1821	11.7	−0.7	1872	−1.02	−2.73
1735	12	0	1822	11.4	−0.9	1873	−1.28	−2.29
1740	12	0	1823	11.1	−1.1	1874	−2.69	−1.97
1745	13	0	1824	10.6	−1.2	1875	−3.24	−2.05
1750	13	0	1825	10.2	−1.4	1876	−3.64	−1.47
1755	14	0	1826	9.6	−1.5	1877	−4.54	−1.32
1760	15	0	1827	9.1	−1.5	1878	−4.71	−1.12
1765	16	0	1828	8.6	−1.5	1879	−5.11	−0.67
1770	16	0	1829	8.0	−1.5	1880	−5.40	−0.35
1775	17	0	1830	7.5	−1.4	1881	−5.42	−0.13
1780	16.9	0.2	1831	7.0	−1.2	1882	−5.20	−0.04
1781	16.9	0.2	1832	6.6	−1.0	1883	−5.46	−0.28
1782	17.0	0.2	1833	6.3	−0.8	1884	−5.46	−0.33
1783	17.1	0.1	1834	6.0	−0.6	1885	−5.79	−0.15
1784	17.1	0.1	1835	5.8	−0.4	1886	−5.63	−0.15
1785	17.1	0.1	1836	5.7	−0.3	1887	−5.64	+0.03
1786	17.1	0.0	1837	5.6	−0.1	1888	−5.80	−0.14
1787	17.1	−0.1	1838	5.6	0.0	1889	−5.66	−0.22
1788	17.1	−0.2	1839	5.6	0.1	1890	−5.87	−0.30
1789	17.0	−0.3	1840	5.7	0.2	1891	−6.01	−0.62
1790	16.9	−0.4	1841	5.8	0.3	1892	−6.19	−0.48
1791	16.7	−0.5	1842	5.9	0.4	1893	−6.64	−0.32
1792	16.5	−0.7	1843	6.1	0.4	1894	−6.44	0.10
1793	16.2	−0.8	1844	6.2	0.4	1895	−6.47	0.58
1794	15.9	−0.9	1845	6.3	0.4	1896	−6.09	1.17
1795	15.6	−1.0	1846	6.5	0.4	1897	−5.76	1.88
1796	15.2	−1.0	1847	6.6	0.4	1898	−4.66	2.40
1797	14.8	−1.0	1848	6.8	0.4	1899	−3.74	2.85
1798	14.4	−1.0	1849	6.9	0.4	1900	−2.72	3.14
1799	14.1	−1.0	1850	7.1	0.4	1901	−1.54	3.46
1800	13.7	−0.9	1851	7.2	0.3	1902	−0.02	3.69

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TABLE 3 (*continued*)

year	ΔT/s	Δ l.o.d./ms	year	ΔT/s	Δ l.o.d./ms	year	ΔT/s	Δ l.o.d./ms
1903	1.24	3.69	1929	24.08	−0.36	1955	31.07	0.90
1904	2.64	3.67	1930	24.02	−0.28	1956	31.35	0.97
1905	3.86	3.43	1931	24.00	−0.11	1957	31.68	1.11
1906	5.37	3.42	1932	23.87	−0.10	1958	32.18	1.26
1907	6.14	3.54	1933	23.95	−0.04	1959	32.68	1.31
1908	7.75	3.61	1934	23.86	−0.08	1960	33.15	1.25
1909	9.13	3.70	1935	23.93	−0.06	1961	33.59	1.21
1910	10.46	3.73	1936	23.73	0.06	1962	34.00	1.27
1911	11.53	3.81	1937	23.92	0.11	1963	34.47	1.45
1912	13.36	3.89	1938	23.96	0.35	1964	35.03	1.74
1913	14.65	3.83	1939	24.02	0.60	1965	35.73	2.03
1914	16.01	3.37	1940	24.33	0.96	1966	36.54	2.25
1915	17.20	3.03	1941	24.83	1.19	1967	37.43	2.38
1916	18.24	2.83	1942	25.30	1.29	1968	38.29	2.48
1917	19.06	2.60	1943	25.70	1.32	1969	39.20	2.57
1918	20.25	2.12	1944	26.24	1.38	1970	40.18	2.69
1919	20.95	1.99	1945	26.77	1.42	1971	41.17	2.84
1920	21.16	1.54	1946	27.28	1.38	1972	42.22	2.96
1921	22.25	1.48	1947	27.78	1.33	1973	43.37	2.97
1922	22.41	1.50	1948	28.25	1.28	1974	44.48	2.89
1923	23.03	1.05	1949	28.71	1.23	1975	45.47	2.81
1924	23.49	0.96	1950	29.15	1.18	1976	46.46	2.77
1925	23.62	0.90	1951	29.57	1.13	1977	47.52	2.82
1926	23.86	0.70	1952	29.97	1.08	1978	48.53	2.80
1927	24.49	0.38	1953	30.36	1.03	1979	49.59	2.62
1928	24.34	−0.03	1954	30.72	0.95	1980	50.54	2.30

dependent on the nature and duration of the decade fluctuations. We shall thus adopt the nearest century year (A.D. 1800) as our standard epoch at which the U.T. day is assumed to be equal to 86400 SI seconds.

5.2. *The mediaeval period*: A.D. 500–1600

Timed observations of lunar and solar eclipse contacts (as measured by Arab astronomers) lead to specific values of ΔT, whereas untimed observations of total and near-total solar eclipses (mainly recorded in Europe) indicate a range of ΔT to satisfy them. For an untimed sighting of the last kind, the range of ΔT may be wide but its limits are sharp, corresponding to the high accuracy with which an observer, using only the unaided eye, can decide whether or not the Sun completely disappears (or leaves a full ring of sunlight at an annular eclipse). The individual ΔT values and ΔT ranges determined from mediaeval observations are shown in figure 6 (there are no useful records between A.D. 500 and 700 so that the first year shown on the figure is the latter). Crosses denote Arabian timings, equal weights being assumed. Vertical bars indicate untimed observations. In only two cases (A.D. 761 and 1567) is the belt of totality sufficiently narrow, or steep relative to the equator, for both ΔT limits to be represented. On all other selected dates, one limit is redundant; this is denoted by an arrow head. In A.D. 1004, an eclipse was observed to be partial rather than central (annular). Hence a rather narrow range of ΔT values – around 200 s – corresponding to the central eclipse is excluded; all other values are acceptable.

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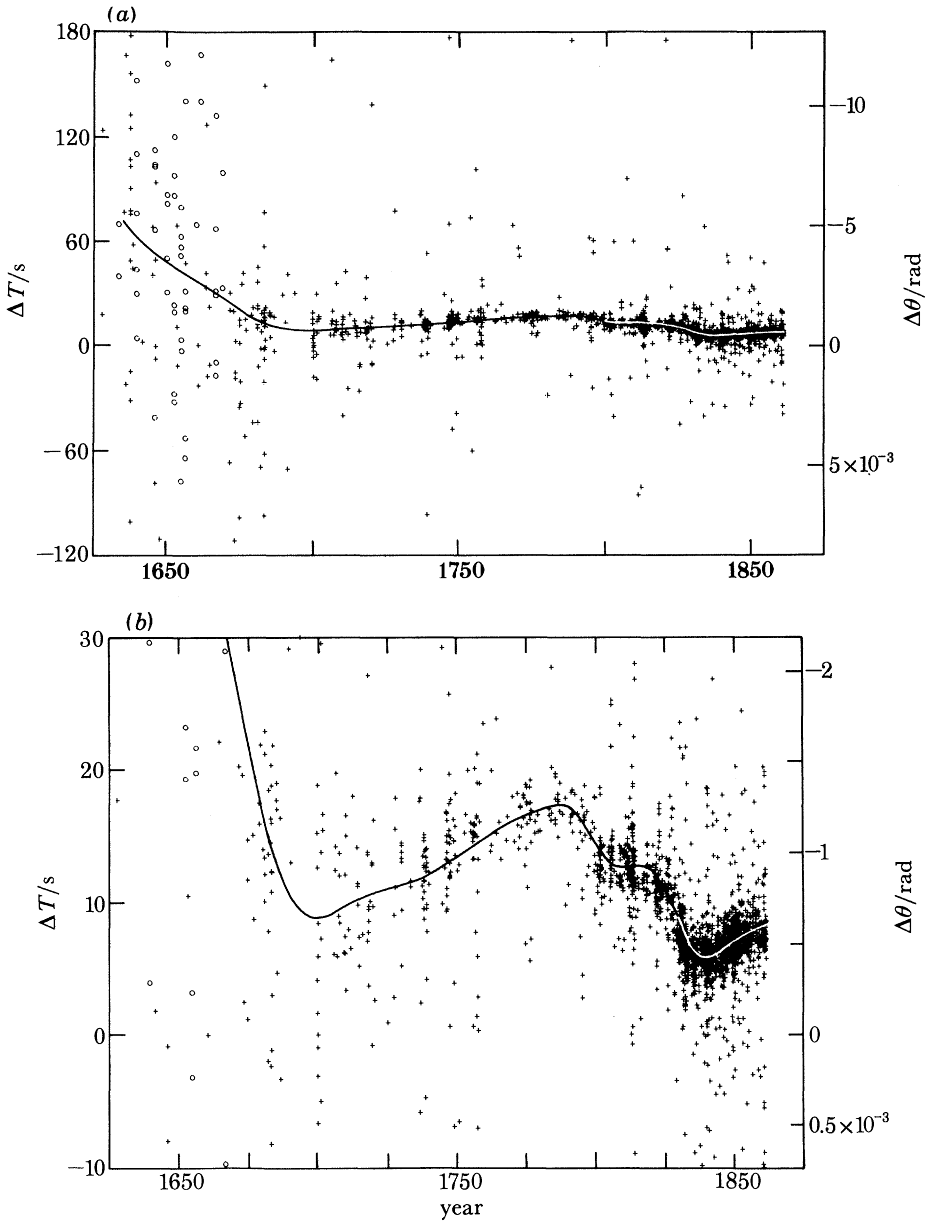

FIGURE 2. (*a*) Values of ΔT between A.D. 1620 and 1860 as derived from individual timings of occultations (crosses) and 4th contacts for solar eclipses (open circles). (*b*) As in (*a*) but on an enlarged scale to show fluctuations after about A.D. 1800 in more detail.

By assuming that ΔT may be approximated by a parabola over the last thousand years or so, the Arabian timings enable a useful mean value of c, the coefficient of t^2, to be obtained. For each individual ΔT result, we have

$$c = (\Delta T)/t^2,$$

where t is time in centuries from A.D. 1800. The mean value of c from the Arabian data is 25.5 ± 1.1 s/cy^2, which leads to a mean result for ΔT at epoch A.D. 948 of 1850 ± 80 s. The untimed data do not provide any such reference value, but they allow the general trend in ΔT to be traced over most of the mediaeval period.

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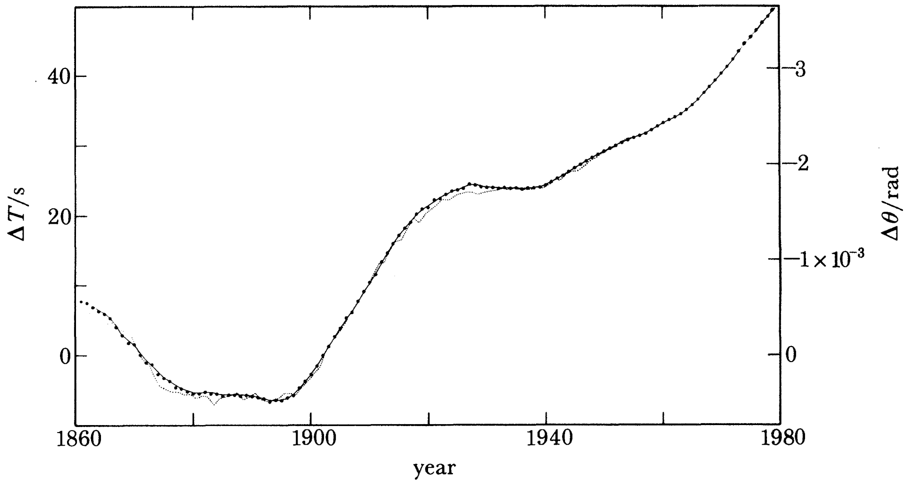

FIGURE 3. Annual solutions (shown as dots) for ΔT in the period A.D. 1861 to 1980. Brouwer's (1952) values, adjusted to $\dot{n} = -26''/\text{cy}^2$, are joined by dotted lines.

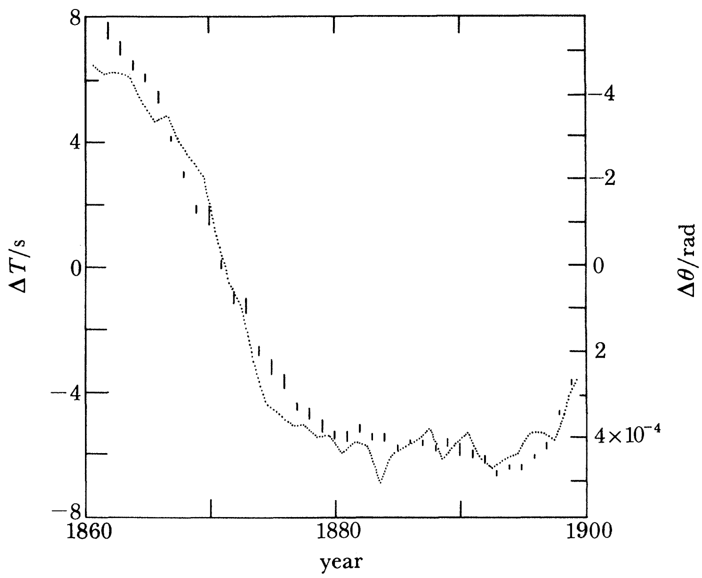

FIGURE 4. Annual solutions and standard error bars for ΔT between A.D. 1861 and 1900. Brouwer's modified values are joined by dotted lines.

5.3. *Ancient observations*: 700 B.C. *to* A.D. 500

In the period 700 B.C. to A.D. 500, we have three independent sets of data: (i) timed Babylonian observations of lunar eclipses; (ii) untimed sightings of total and near-total solar eclipses (Chinese and Babylonian); (iii) Babylonian observations of the Moon rising and setting while eclipsed. The various ΔT values or ranges determined from these observations are shown in figure 7. This covers the interval from 700 B.C. to A.D. 200; there are no useful observations between A.D. 200 and 500.

The Babylonian timed data lead to individual values of ΔT. In figure 7, we have distinguished two sets of measurements: those for which the timed interval (in relation to sunrise or sunset) was less than 20 *uš* or 80 min, and those for which the interval was longer than 20 *uš*.

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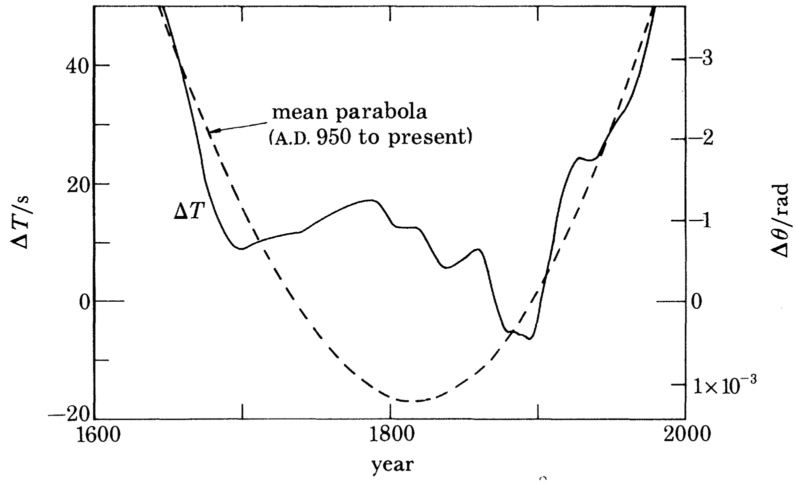

FIGURE 5. Superposition of mean parabola (dashed) since mediaeval times on telescopic ΔT curve. The epoch of best fit is close to A.D. 1800.

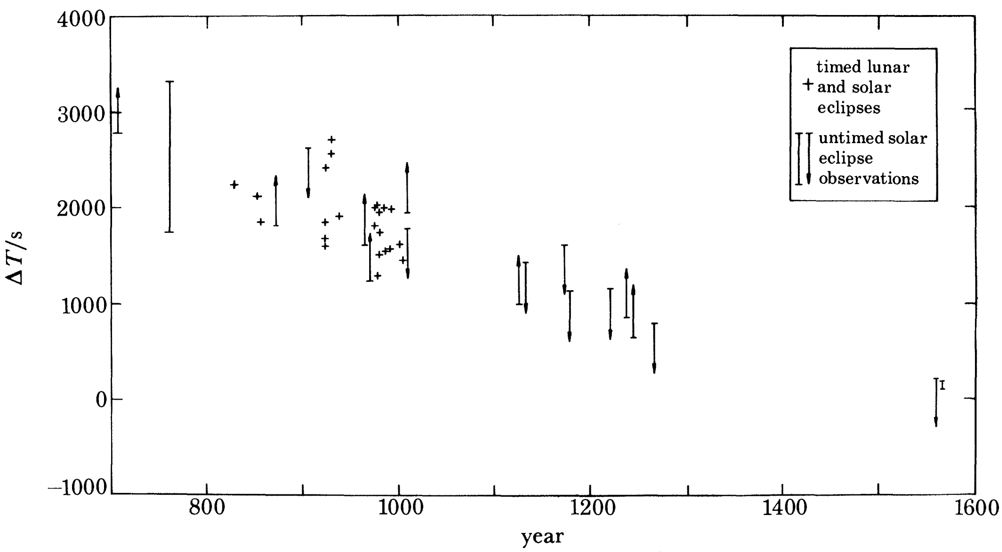

FIGURE 6. ΔT solutions for the mediaeval period (A.D. 700 to 1600) derived from both timed and untimed observations of eclipses.

In a short interval of time, clock drift seems likely to be small, whereas for some of the longer intervals (in the region of 100 *uš*, or more than 6 h) there is a distinct possibility of serious timing errors. We have distinguished these two sets of observations by filled circles and open circles respectively.

Untimed total solar eclipses are indicated by vertical bars in figure 7, following the convention adopted in figure 6. Observations of lunar eclipses near moonrise or moonset lead to single definite limits for ΔT (represented by dotted vertical lines in the diagram).

5.4. *General ΔT plot*

A ΔT plot over the whole of the historical period is given in figure 8. Times are on a logarithmic scale owing to the large range of ΔT. In order to avoid negative numbers, +30 s has been added to all values. Lambeck (1980, equation 10.5.2) calculated the tidal acceleration

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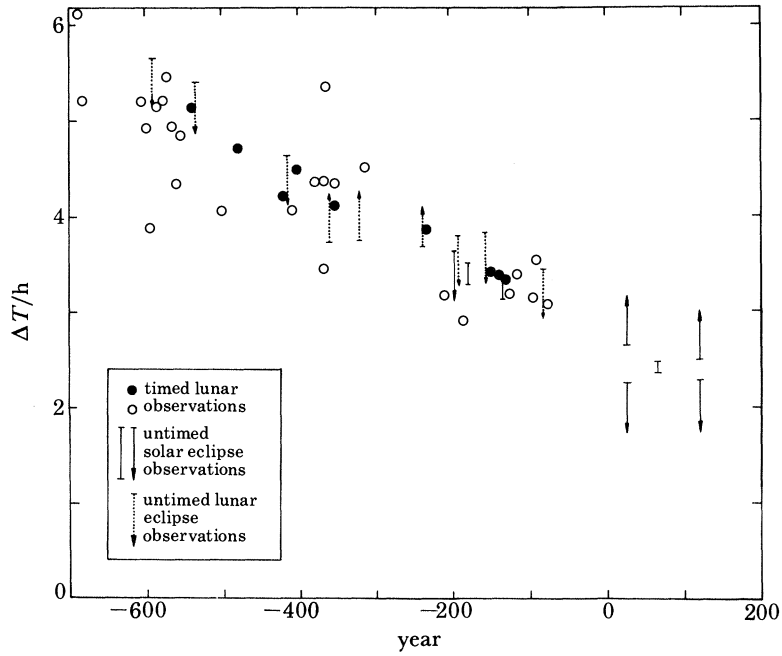

FIGURE 7. ΔT solutions for the ancient period (700 B.C. to A.D. 200) derived from both timed and untimed eclipse observations.

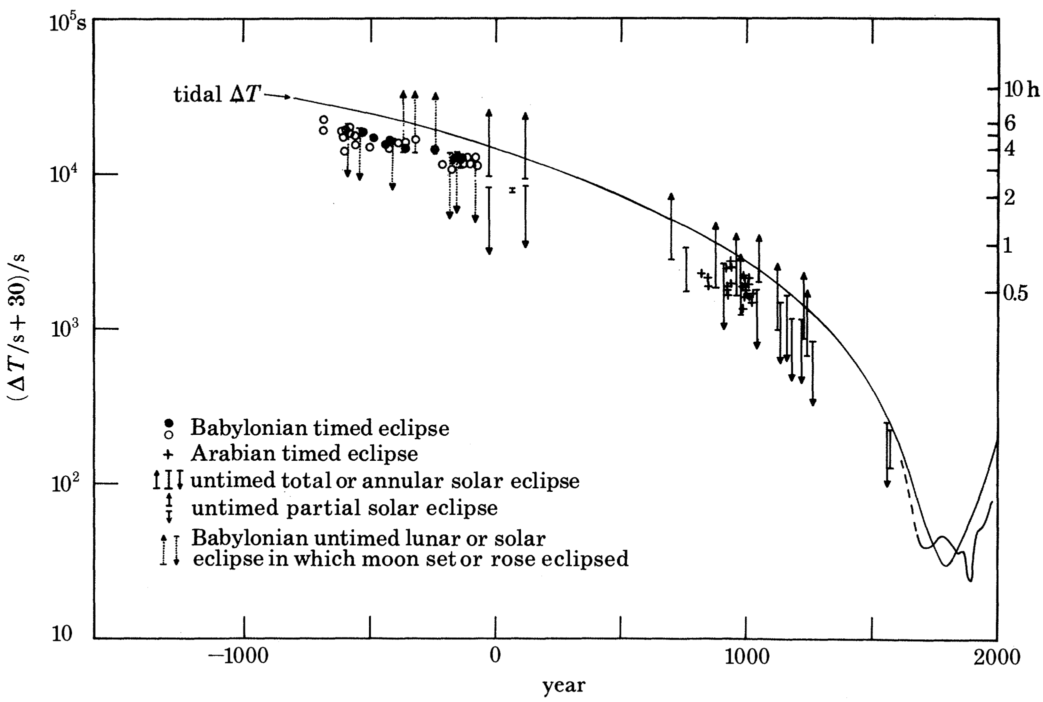

FIGURE 8. Plot of ΔT on a logarithmic scale for the period 700 B.C. to A.D. 1980. 30 s has been added to each value to make all of them positive. The continuous line shows the computed tidal deceleration of the Earth.

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of the Earth's spin in terms of the lunar orbital acceleration $\dot{n}$ on the basis of conservation of angular momentum in the Earth–Moon system. He allowed for both lunar and solar tides – in the solid body of the Earth, oceans and atmosphere – and found

$$\dot{\omega}_t = 51\dot{n}''/\text{cy}^2,$$

i.e.

$$(\dot{\omega}_t/\omega) = 1.07\times10^{-11}\,\dot{n}\ \text{a}^{-1}. \qquad (7)$$

With $\dot{n} = -26''/\text{cy}^2$,

$$(\dot{\omega}_t/\omega) = -27.8\times10^{-11}\ \text{a}^{-1}.$$

which is equivalent to (see table 2)

$$\mathrm{d}(\text{l.o.d.})_t/\mathrm{d}t = +2.40\ \text{ms/cy}$$

and

$$(\Delta T)_t = 43.8\,t^2\ s.$$

This tidal deceleration is represented in figure 8 by a smooth curve. It is evident from the diagram that throughout the ancient and mediaeval periods the tidal deceleration is inconsistent with observation, indicating the presence of a non-tidal residual acceleration.

It is to be noted that no *single* parabola will fit both the ancient and mediaeval data and still have its apex near A.D. 1800 (or pass roughly through the telescopic data, as in figure 5). Since the data tend to group into three discrete periods, we make the simplest assumption that one acceleration applies to the range 390 B.C. (the Babylonian mean) to A.D. 950 (the Arabian mean) and another from A.D. 950 to 1800. This is not to imply that an abrupt change occurred around A.D. 950. The change could have been quite gradual and might have occurred anywhere between about A.D. 200 and 950. There are insufficient data to answer this point.

6. Results for l.o.d. and rate of change in l.o.d.

6.1. *The telescopic period*: A.D. 1620–1980

Changes in the l.o.d. during this period were calculated from the first derivative of ΔT as follows. Before 1861 the derivative of the spline curves was evaluated at yearly intervals. From 1861 onwards the derivatives were calculated annually by applying a five-point quadratic convolute to the annual mean values of ΔT (see Morrison 1979*a*). Annual (or earlier, five-yearly) values of the change in l.o.d. relative to the standard day of 86400 SI seconds are given in table 3, alongside the appropriate ΔT values.

Figure 9 shows the changes in l.o.d. during the telescopic period. Because of the inaccuracy of observations before about A.D. 1780, only the general form of the early part of the curve can be traced. Reference to the subsidiary plot below the main diagram in figure 9 (giving estimates of the standard deviation (σ)) indicates that after about A.D. 1780 the form of the curve representing changes in l.o.d. is fairly well established, including the fine structure after about A.D. 1870. The right scale in figure 9 is marked directly in terms of changes in the angular speed of rotation of the Earth. The dotted straight line intersecting the main curve indicates the average rate of change in l.o.d. between the period of the Arabian observations (*ca.* A.D. 950) and A.D. 1800; this is +1.4 ms/cy, which is equivalent to the mean value of 25.5 s/cy² for c derived in §5.2.

6.2. *The mediaeval period*: A.D. 500–1600

For the data in this period, Δ l.o.d. values were deduced from the observed ΔT values by using the expression:

$$\Delta\,\text{l.o.d.} = 2\Delta T/36\,525(t-18.00)\ \text{s}. \qquad (8)$$

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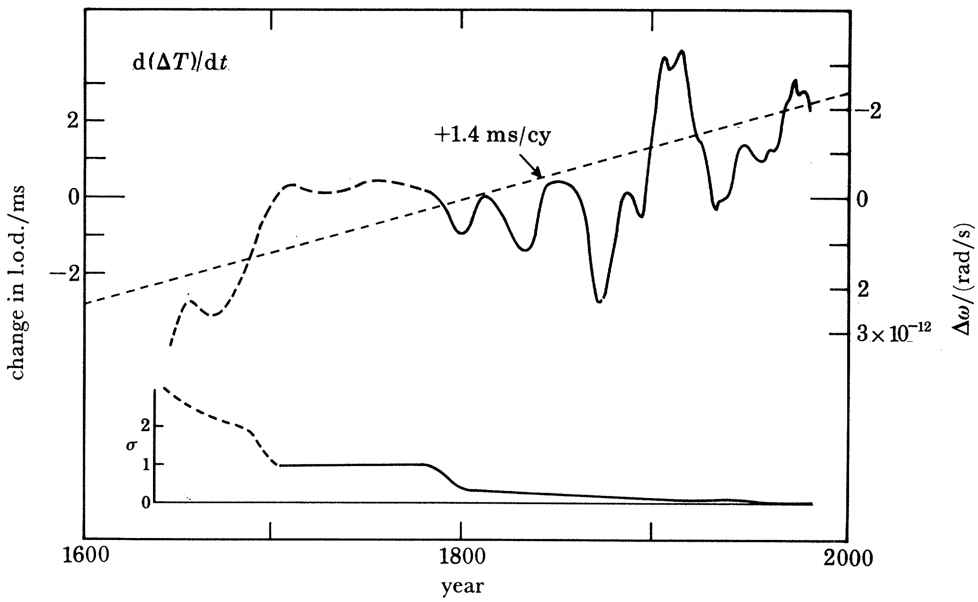

FIGURE 9. Changes in l.o.d. (length of day) for the period A.D. 1620 to 1980. The dotted line represents the average rate of change in l.o.d. since mediaeval times. An estimate of the standard deviation (σ) is also shown.

This assumes a constant acceleration of the Earth's spin between the epoch of each observation and A.D. 1800. The result from the timed Arabian observations with $\Delta T = +1850\ \text{s} \pm 80\ \text{s}$ from §5.2 is:

$$\Delta\,\text{l.o.d.} = -11.9 \pm 0.5\ \text{ms at A.D. } 948.$$

The average rate of lengthening of the day between A.D. 948 and 1800 is thus $+1.40 \pm 0.04$ ms/cy.

6.3. *The ancient period*: 700 B.C. *to* A.D. 500

To deduce changes in the l.o.d. at this period, a constant acceleration of the Earth's rotation was assumed between the epoch of the observations and the mean epoch of the Arabian data (A.D. 948). Each Δ l.o.d. value was calculated from the expression

$$\Delta\,\text{l.o.d.} = 2(\Delta T - 1850)/36525(t - 9.48) + 0.0119\ \text{s} \tag{9}$$

with the mean values for ΔT and Δ l.o.d. at A.D. 948 of $+1850$ s and -11.9 ms. The average epoch for the forty or so Babylonian timed observations is 390 B.C. These data yield a mean value for ΔT at this epoch of 15600 ± 350 s which gives a result for Δ l.o.d. of -44.4 ± 1.0 ms at 390 B.C. The average rate of lengthening of the day between 390 B.C. and A.D. 948 is thus $+2.43 \pm 0.07$ ms/cy.

6.4. *Graphical representation of ancient and mediaeval data*

The ancient and mediaeval results for the change in l.o.d., in relation to the reference value, are plotted in figure 10 along with the modern curve. The same convention is used as in earlier figures. In this diagram, the solid straight line represents the computed tidal rate in the l.o.d. due to Lambeck (1980), i.e. $+2.40$ ms/cy assuming $\dot{n} = -26''/\text{cy}^2$. The dotted straight lines having slopes 2.4 ms/cy and 1.4 ms/cy denote the average rates of change in l.o.d. between

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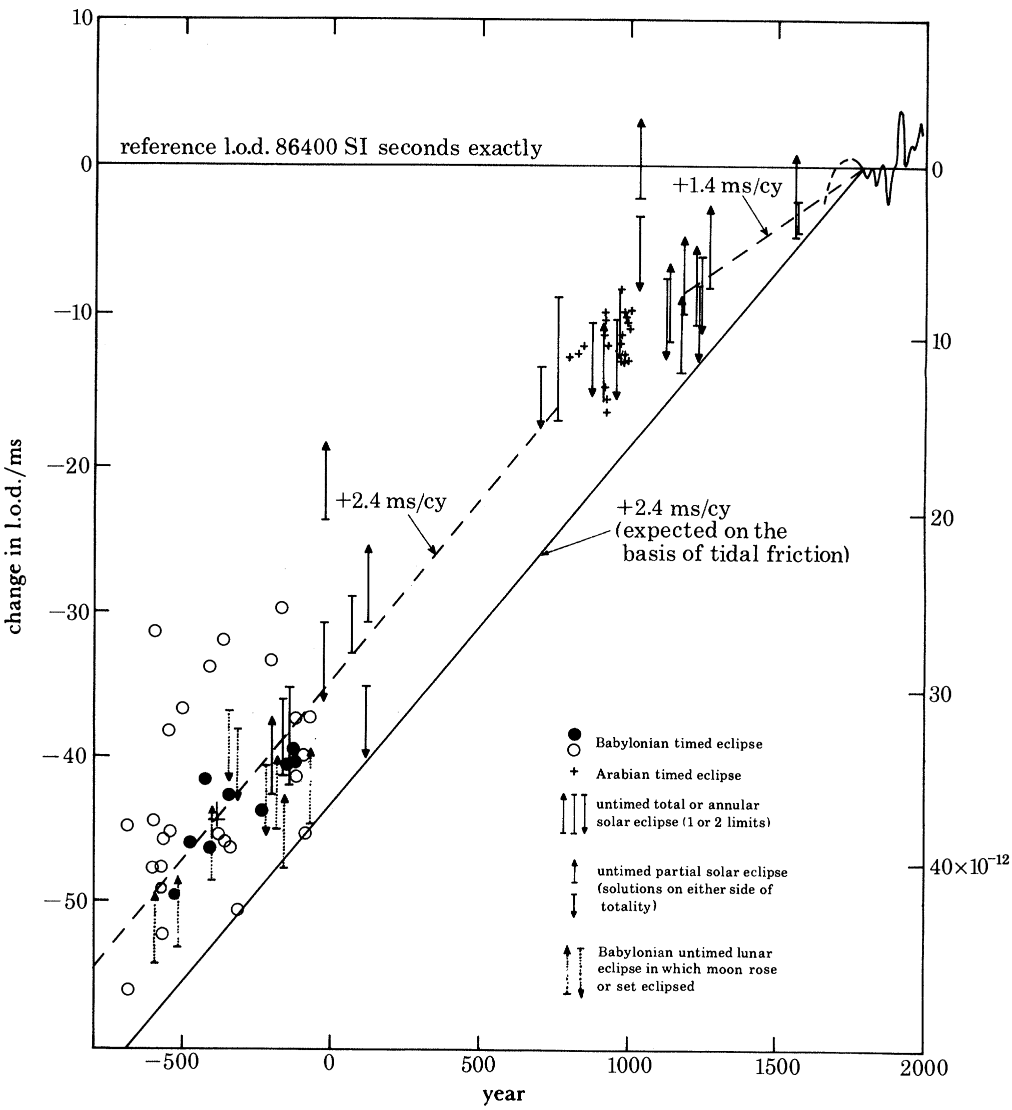

FIGURE 10. Changes in l.o.d. for the period 700 B.C. to A.D. 1980. The expected rate of change due to tidal braking is shown as a continuous line. The mean observed rates of change are represented by dashed lines.

(i) the mean epoch of the timed Babylonian observations (390 B.C.) and that of the Arabian data, and (ii) between A.D. 948 and 1800. We have only shown sections of these dotted lines to emphasize that we do not envisage a sudden change around A.D. 950.

7. Non-tidal changes in the l.o.d.

The non-tidal changes in the l.o.d. are found by subtracting the tidal component of +2.4 ms/cy from the observed changes. By subtracting (7) from (4) we find the non-tidal component to be

$$(\dot{\omega}/\omega)-(\dot{\omega}_t/\omega) = -0.45\times10^{-11}\dot{n}-(\nu-\nu')/(n-n'). \qquad (10)$$

The error in the non-tidal component due to an error of $\pm 1''/\text{cy}^2$ in $\dot{n}$ is therefore $\pm 0.45\times10^{-11}$ per year, which is equivalent to ± 0.04 ms/cy in the rate of change of l.o.d. This error will be further increased by any error in the coefficient of $\dot{n}$ in (7).

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7.1. *The telescopic period*: A.D. 1620–1980

The modern non-tidal changes in l.o.d. are shown in figure 11. The data since A.D. 1780 reveal fluctuations in l.o.d. of about 4 ms amplitude. A power spectrum analysis of the data since A.D. 1861 shows that most of the power occurs around the 30-year periodicity. The torques attained a magnitude of some 10^{18} N m (Morrison 1979*a*). It is possible that similar fluctuations occurred during the interval A.D. 1620–1780, but the resolution here is too poor for such changes to be detected; only the general trend is apparent.

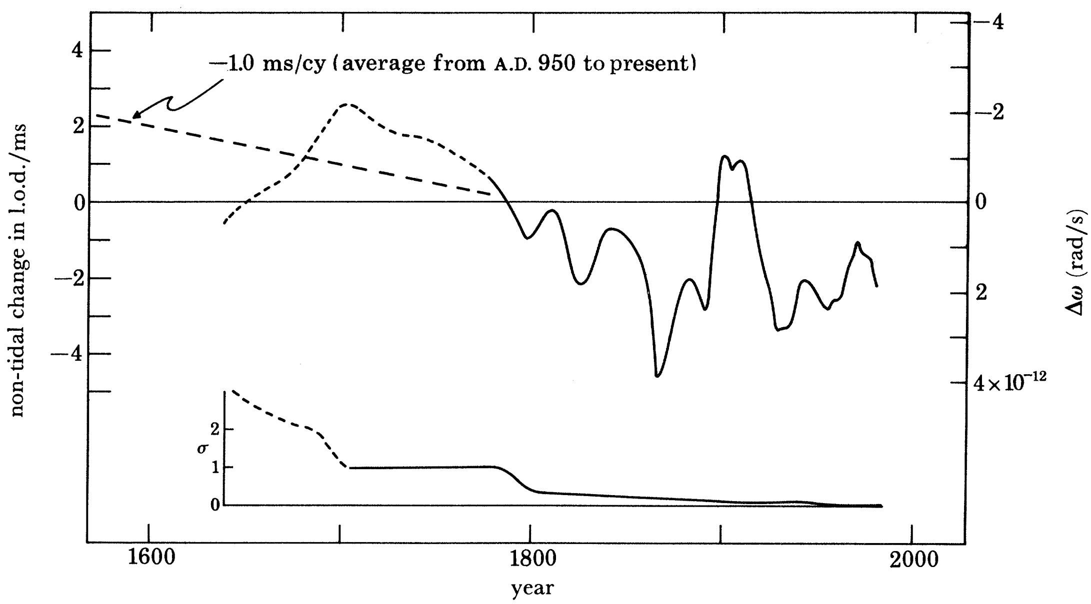

FIGURE 11. Non-tidal changes in l.o.d. between A.D. 1620 and 1980. An estimate of the standard deviation (σ) is also shown.

7.2. *The pre-telescopic period*: 700 B.C. *to* A.D. 1600

A plot of the long-term tidal variations in l.o.d. for the whole of the historical period is shown in figure 12. Two mean rates are shown by dotted lines. The Arabian timings indicate an average rate of decrease in l.o.d. due to non-tidal causes of about 1.0 ms/cy since the mean epoch A.D. 948; at this epoch, the l.o.d. due to non-tidal causes was 8.5 ms longer than the reference l.o.d. The Babylonian data indicate no significant changes in l.o.d. due to causes other than tides in ancient times. However, as already pointed out, we cannot ascertain from the existing observations the precise behaviour of the non-tidal component in the Earth's rotation in the pre-telescopic past. Nevertheless, there is ample evidence of variation on the timescale of millennia.

CONCLUSION

1. *Tidal deceleration*

The value for the lunar tidal acceleration that we have adopted in this work is $\dot{n} = -26''/\text{cy}^2$. Lunar and solar tidal braking is shown to be the dominant long-term mechanism reducing the Earth's rate of rotation (see figure 10). We find that, owing to tidal action alone:

$$\text{rate of change in l.o.d., d(l.o.d.)}/\mathrm{d}t = +2.40 \text{ ms/cy}$$

$$\equiv \text{spin acceleration, } \dot{\omega} = -6.4\times10^{-22} \text{ rad/s}^2;$$

$$\text{torque, } C\dot{\omega} = -4.6\times10^{+16} \text{ N m} \quad (C = 7.2\times10^{37} \text{ kg m}^2).$$

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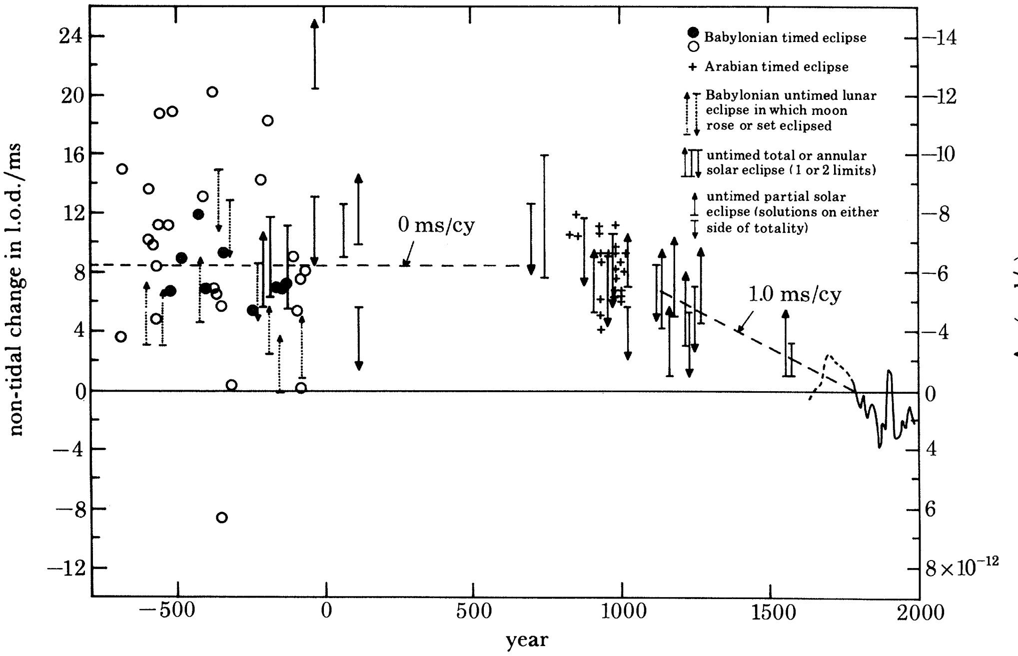

FIGURE 12. Non-tidal changes in l.o.d. for the period 700 B.C. to A.D. 1980. The mean observed rates of change are represented by dashed lines.

2. *Expressions for* ΔT

(*a*) Detailed values for ΔT from A.D. 1630 to 1980 are given in table 3.

(*b*) A parabolic representation of the data from A.D. 948 (the mean epoch of the Arabian data) to A.D. 1600 is given by

$$\Delta T/\mathrm{s} = +25.5\,t^2,$$

where t is time in centuries measured from A.D. 1800. This result is equivalent to a rate of lengthening of the day of +1.4 ms/cy.

(*c*) A parabolic representation of the data from 390 B.C. (the mean epoch of the Babylonian data) to A.D. 948 is given by

$$\Delta T/\mathrm{s} = +1850-435\,\tau+44.3\,\tau^2,$$

where τ is measured in centuries from A.D. 948. With the origin at A.D. 1800, the corresponding expression is

$$\Delta T/\mathrm{s} = +1360+320\,t+44.3\,t^2.$$

The equivalent rate of lengthening of the day is +2.4 ms/cy.

3. *Non-tidal changes*

Non-tidal variations occur on timescales ranging from decades to millennia.

(*a*) *Decade changes: since* A.D. 1780 (figure 11)

Change in l.o.d., $|\Delta\,\mathrm{l.o.d.}|_{\max} \approx 4$ ms in 7 years;

change in rate of rotation, $|\Delta\omega|_{\max} \approx 3\times10^{-12}$ rad/s in 7 years;

change in angular momentum, $|C\Delta\omega|_{\max} \approx 2.4\times10^{26}$ kg m²/s in 7 years;

acceleration, $|\dot{\omega}|_{\max} \approx 1.4\times10^{-20}$ rad/s²;

torque, $C|\dot{\omega}|_{\max} \approx 10^{18}$ N m.

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(*b*) *Long-term changes: since* A.D. 950 (figure 12)

Change in l.o.d., Δ l.o.d. $= -8$ ms between A.D. 950 and present;
change in rate of rotation, $\Delta\omega = +7 \times 10^{-12}$ rad/s between A.D. 950 and present;
change in angular momentum, $C\Delta\omega = +5 \times 10^{+26}$ kg m^2/s between A.D. 950 and present;
average acceleration, $\dot{\omega} = +2.7 \times 10^{-22}$ rad/s^2 since A.D. 950.

(*c*) *Long-term changes*: *before* A.D. 950 (figure 12)

On our interpretation of the data, we find no appreciable non-tidal component before A.D. 950. We do not suggest a sudden change at this epoch; the observations analysed do not allow either the form of the change or its mean epoch to be determined with any degree of confidence. Analysis of the large number of recorded Chinese observations of occultations of planets and bright stars by the Moon made during the first millennium A.D. may help to clarify this issue. Although these observations are largely untimed, they are so numerous (numbering about 1000 individual records) that we might expect a fair number of grazing occultations; these would set useful limits to ΔT. This should provide improved resolution in changes in the l.o.d. before A.D. 950.

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Discussion

G. J. Whitrow (*Mathematics Department, Imperial College, London U.K.*). I should like to ask Dr Stephenson whether he thinks it possible to bridge the gap between the relevant Seleucid–Babylonian astronomical records of the last few centuries B.C. and the mediaeval monastery records by consulting the astronomical records left by the Mayas of Central America. Their civilization reached its highest stage between 300 and 900 A.D. They devised a calender that was even more accurate than our present Gregorian calendar. It is possible that the Dresden Codex, which contains Mayan data concerning lunar and solar eclipses and also the planets, may provide further information bearing on changes in the Earth's rate of rotation in the historical past.

F. R. Stephenson. I have doubts whether it will prove possible to bridge the gap in question with Maya eclipse records. Judging from the work of the late J. E. S. Thompson (see, for example, *Phil. Trans. R. Soc. Lond.* A **276**, 83–98 (1974)), the Dresden codex does not record observed eclipses but rather gives a table of data for eclipse *prediction*. At present, I am unaware of any Maya texts that actually relate to eclipse observations.

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